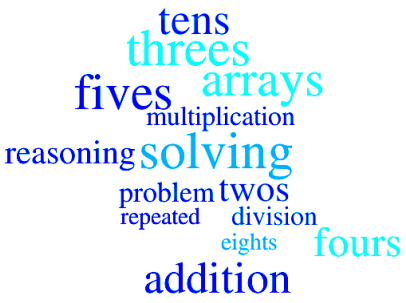
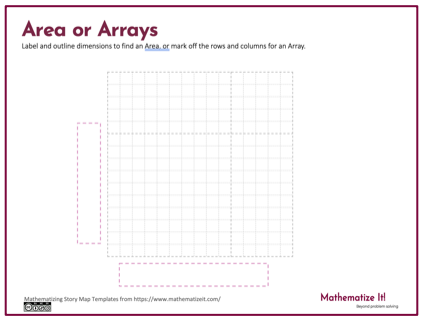
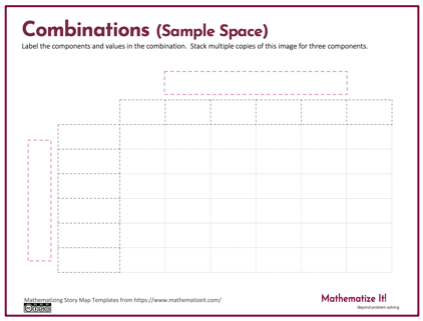


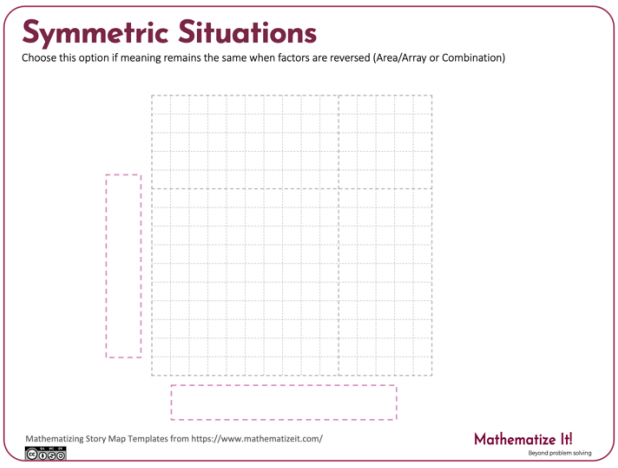
Symmetric Multiplication and Division Summary

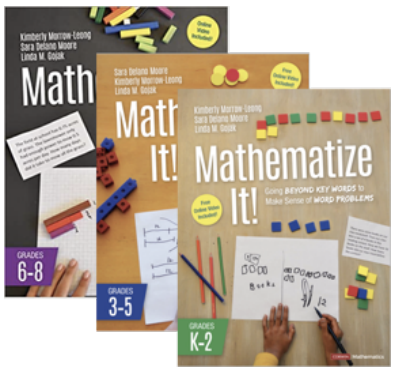
Making sense of word problems with mathematical comprehension & operation sense.

Mathematizing Story Maps









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Sara Delano Moore and Kimberly Morrow-Leong
Mathematizing Story Map Templates from https://www.mathematizeit.com/

Teacher Notes
Mathematizing Story Maps
Multiplication & Division Summary

Problem Types
There are four different types of word problems or problem situations that are represented by multiplication or division: Equal Groups, Multiplicative Comparison, Area/Array, and Combinatorics. A Mathematizing Story Map for each, including practice problems, is available on our website. Now we summarize all four as two separate groups, each with similar features and characteristics.

Asymmetric
The problems in the first group are situations where the two factors do different jobs. One factor represents a quantity (the starting value or the size of the group) while the other factor acts on that quantity (making copies or scaling). This general class of problems is a precursor to the study of ratio and rate in the middle grades.

Students' first introduction to multiplication and division is often through equal groups situations. In these problem situations, the factors are either the group size or the number of groups. The same is true with the Multiplicative Comparison problem situation: one factor tells how many times bigger while the other tells what is being measured.

As with other problem situations, any of the three values in the equation may be unknown. All of these problem types are called Asymmetric problem types because the factors have a different role in the situation.

Symmetric
In symmetric problems, the two factors are more alike than different, so they can be interchanged with no change in the meaning of a situation. Arrays, areas, and counting problems are good examples. We also know a problem situation is asymmetric because the product represents something different than the factors do.

An Area/Array situation may be better thought of as two measures of linear dimension (feet, yard, cm, m, km) that multiply to make a new unit of measure: a *square* unit. Area is measured with squares. An array is similar because a new unit is named where a row and column meet, yet it isn't called either of those names: it's a cell, a square, etc..

A Combinatorics situation describes finding the total number of possible outcomes given a specific set of inputs. For example, ice cream and a topping (ingredients) combine to create sundaes. In this situation, each factor does the same job of representing one component of a possible outcome, known as the Sample Space.

As with other problem situations, any of the three values in the equation may be unknown. These problem situations are described as symmetric because the factors are more alike than different.

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Multiplication & Division Problem Situations				
Asymmetric Situations	**Equal Groups (Ratio/Rate)**	Product Unknown	Number of Groups (Multiplier) Unknown	Group Size (Measure) Unknown
	Multiplicative Comparison	Resulting Value Unknown	Scale Factor Unknown	Original Value Unknown
Symmetric Situations	**Area/Array**	Product Unknown	One-Dimension Unknown	Both Dimensions Unknown
	Combinatorics	Sample Space (Total Outcomes) Unknown	One Factor Unknown	Both Factors Unknown

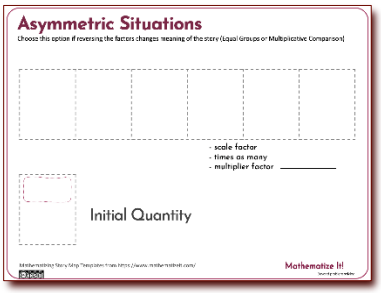
Missing Element

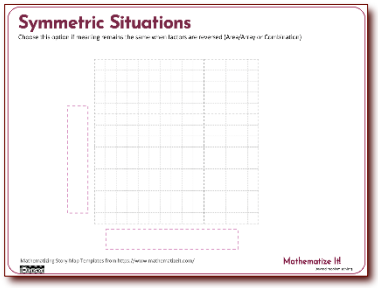
In an unsolved problems situation one element is always unknown. For the Equal Groups situations, the three possibilities are the size of one group, the number of groups, or the total. In a Multiplicative Comparison problem situation, the three elements are the starting value, the scale factor, and the resulting value.

In a symmetric problem situation, the missing elements could be one of the factors or the product in an Area/Array or combinatorics situation as well.

The Mathematizing Story Maps

The two story maps shared this week are more general than the ones specific to each problem type. One story map shows the starting value (or a single group) and then leaves space for scaling (or multiple copies). The other story map uses a grid to show how two factors can be combined into a new unit.





Common Structure and Strategies

Begin by reading the story at least once with your class. Talk about the story and ask students to make sense of the events in the narrative as you would for any story. Then encourage students to find the mathematics in the story with questions like these:

- *What are the roles of the factors in this story? Do they do the same job or different jobs?*
- *If the factors do the same work, what are the units for the factors? For the product? How does this help illustrate the situation?*
- *If the factors do different jobs, what is happening in the situation? Are there copies being made or is something being scaled larger (or smaller)?*

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These questions ask students to reason about which story map might best support their thinking. If students start calculating numbers right away, particularly if they are "number-plucking" or randomly doing calculations, refocus their attention on the relationships in the story.

Priya hung a hummingbird feeder outside her window. She read that hummingbirds eat 1.8 times their weight each day. On the first day a hummingbird took 7.2 grams of food. How much does the hummingbird probably weigh?

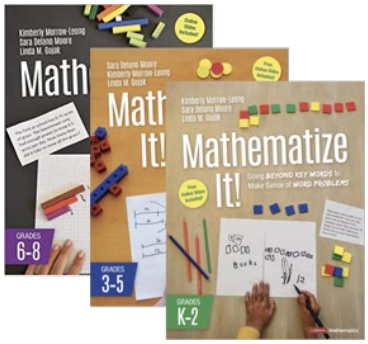
Different Classes of Number

These practice problems represent some of the many multiplication and division problems situations in contexts involving fractions and decimals. Work with the quantities under 100 in previous units allows students to focus on the structure of the problem situation without being bogged down in the computation. In these problems, students are now more familiar with the structures and can work on more challenging number classes.

Modifying the Problems

Each problem is identified when it is presented with a possible solution. Feel free to adapt the problems to provide a full range of examples for your students.

For example, the problem shown above represents a multiplicative comparison with the initial quantity unknown. If students need to practice situations where the resulting quantity is unknown instead, use the same situation and rewrite the problem as this: "The hummingbird eats 1.8 times its weight each day. If the hummingbird weighs 4 grams, how much does it eat in a day?"



To read more about problem situations and the four operations, check out the ***Mathematize It!*** book series.

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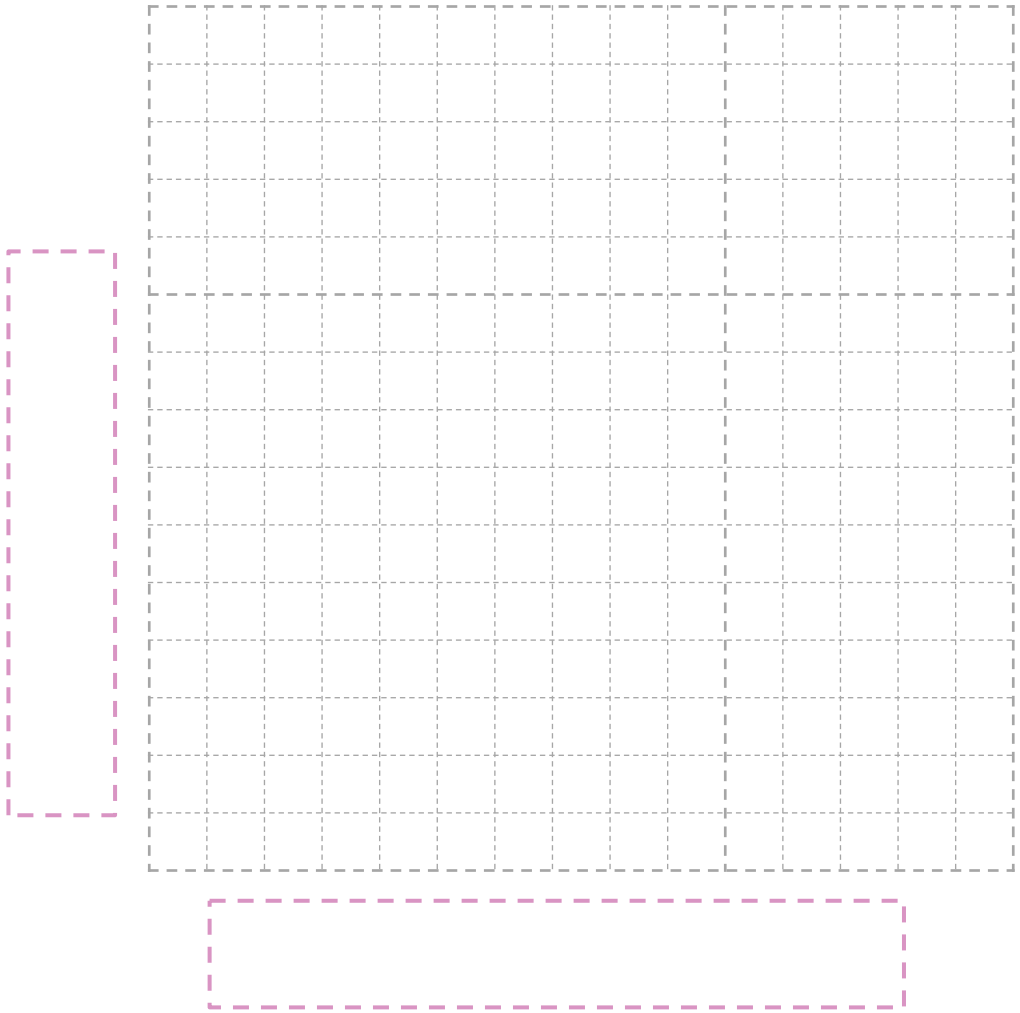
Symmetric Situations

Choose this option if meaning remains the same when factors are reversed (Area/Array or Combination)



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Symmetric Practice Problems

Use objects, pictures, numbers, and words to describe what is happening in each problem. Use a mathematizing story map to record your thinking.

The town is starting a community garden where each family has been assigned a plot that measures 250 m^2 of space in the garden. Each garden plot is shaped like a rectangle and is 12.5 meters wide. How long is each garden?
The class is helping arrange the chairs for an assembly in the gym. They need to seat 275 students and can place 20 chairs in a row. How many rows of chairs do they need?
The game uses three dice and advertises 288 possible combinations. Two of the dice have 6 sides each. How many sides are on the third die?

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Symmetric Practice Problems

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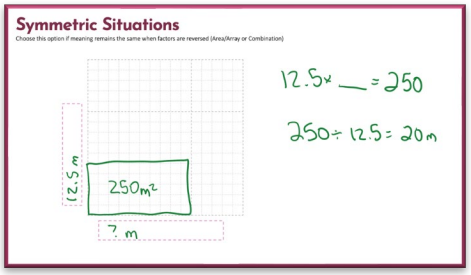

Area/Array, One Dimensions Unknown

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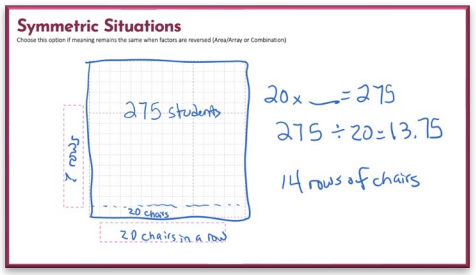

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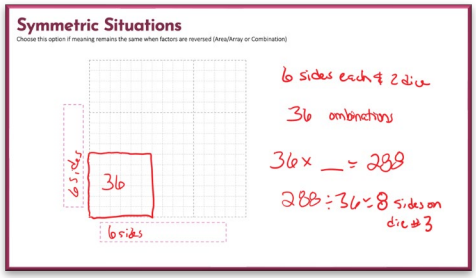

Combinations, One Dimension Unknown

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